

**nag\_corr\_cov (g02bxc)****1. Purpose**

**nag\_corr\_cov (g02bxc)** calculates the Pearson product-moment correlation coefficients and the variance-covariance matrix for a set of data. Weights may be used.

**2. Specification**

```
#include <nag.h>
#include <nagg02.h>

void nag_corr_cov(Integer n, Integer m, double x[], Integer tdx,
                  Integer sx[], double wt[], double *sw, double wmean[], double std[],
                  double r[], Integer tdr, double v[], Integer tdv, NagError *fail)
```

**3. Description**

For  $n$  observations on  $m$  variables a one-pass updating algorithm (see West 1979) is used to compute the means, the standard deviations, the variance-covariance matrix, and the Pearson product-moment correlation matrix for  $p$  selected variables. Suitable weights may be used to indicate multiple observations and to remove missing values.

The quantities are defined by:

(a) The means

$$\bar{x}_j = \frac{\sum_{i=1}^n w_i x_{ij}}{\sum_{i=1}^n w_i} \quad j = 1, \dots, p$$

(b) The variance-covariance matrix

$$C_{jk} = \frac{\sum_{i=1}^n w_i (x_{ij} - \bar{x}_j)(x_{ik} - \bar{x}_k)}{\sum_{i=1}^n w_i - 1} \quad j, k = 1, \dots, p$$

(c) The standard deviations

$$s_j = \sqrt{C_{jj}} \quad j = 1, \dots, p$$

(d) The Pearson product-moment correlation coefficients

$$R_{jk} = \frac{C_{jk}}{\sqrt{C_{jj}C_{kk}}} \quad j, k = 1, \dots, p$$

where  $x_{ij}$  is the value of the  $i$ th observation on the  $j$ th variable and  $w_i$  is the weight for the  $i$ th observation which will be 1 in the unweighted case.

Note that the denominator for the variance-covariance is  $\sum_{i=1}^n w_i - 1$ , so the weights should be scaled so that the sum of weights reflects the true sample size.

**4. Parameters**

**n**

Input: the number of observations in the data set,  $n$ .

Constraint: **n** > 1.

**m**

Input: the total number of variables,  $m$ .

Constraint: **m** ≥ 1.

**x[n][tdx]**

Input: the data  $\mathbf{x}[i-1][j-1]$  must contain the  $i$ th observation on the  $j$ th variable,  $x_{ij}$ , for  $i = 1, \dots, n$ ;  $j = 1, \dots, m$ .

**tdx**

Input: the second dimension of the array  $\mathbf{x}$  as declared in the function from which nag\_corr\_cov is called.

Constraint:  $\mathbf{tdx} \geq \mathbf{m}$ .

**sx[m]**

Input: indicates which  $p$  variables to include in the analysis.

If  $\mathbf{sx}[j-1] > 0$ , the  $j$ th variable is to be included.

If  $\mathbf{sx}[j-1] = 0$ , the  $j$ th variable is not to be included.

If  $\mathbf{sx}$  is set to the null pointer (Integer \*)0 then all variables are included in the analysis, i.e.,  $p = m$ .

Constraint:  $\mathbf{sx}[i] \geq 0$ , for  $i = 1, \dots, m$ .

**wt[n]**

Input: the optional frequency weighting for each observation.  $\mathbf{wt}[i-1]$  contains the weight for the  $i$ th data value. Usually  $\mathbf{wt}[i-1]$  will be an integral value corresponding to the number of observations associated with the  $i$ th data value, or zero if the  $i$ th data value is to be ignored. If  $\mathbf{wt}$  is set to the null pointer (double \*)0 then  $\mathbf{wt}$  is not referenced.

Constraint:  $\mathbf{wt}[i-1] \geq 0.0$ , for  $i = 1, \dots, n$ .

**sw**

Output: the sum of weights if  $\mathbf{wt}$  is not the null pointer, otherwise  $\mathbf{sw}$  contains the number of observations,  $n$ .

**wmean[m]**

Output: the sample means.  $\mathbf{wmean}[j-1]$  contains the mean for the  $j$ th variable.

**std[m]**

Output: the standard deviations.  $\mathbf{std}[j-1]$  contains the standard deviation for the  $j$ th variable.

**r[m][tdr]**

Output: the matrix of Pearson product-moment correlation coefficients.  $\mathbf{r}[j-1][k-1]$  contains the correlation between variables  $j$  and  $k$ , for  $j, k = 1, \dots, p$ .

**tdr**

Input: the second dimension of the array  $\mathbf{r}$  as declared in the function from which nag\_corr\_cov is called.

Constraint:  $\mathbf{tdr} \geq \mathbf{m}$ .

**v[m][tdv]**

Output: the variance-covariance matrix.  $\mathbf{v}[j-1][k-1]$  contains the covariance between variables  $j$  and  $k$ , for  $j, k = 1, \dots, p$ .

**tdv**

Input: the second dimension of the array  $\mathbf{v}$  as declared in the function from which nag\_corr\_cov is called.

Constraint:  $\mathbf{tdv} \geq \mathbf{m}$ .

**fail**

The NAG error parameter, see the Essential Introduction to the NAG C Library.

## 5. Error Indications and Warnings

**NE\_INT\_ARG\_LE**

On entry,  $\mathbf{n}$  must be greater than 1:  $\mathbf{n} = \langle \text{value} \rangle$ .

**NE\_INT\_ARG\_LT**

On entry,  $\mathbf{m}$  must not be less than 1:  $\mathbf{m} = \langle \text{value} \rangle$ .

**NE\_2\_INT\_ARG\_LT**

On entry, **tdx** = *<value>* while **m** = *<value>*.

These parameters must satisfy **tdx** ≥ **m**.

On entry, **tdr** = *<value>* while **m** = *<value>*.

The parameters must satisfy **tdr** ≥ **m**.

On entry, **tdv** = *<value>* while **m** = *<value>*.

These parameters must satisfy **tdv** ≥ **m**.

**NE\_NEG\_WEIGHT**

On entry, at least one of the weights is negative.

**NE\_NEG\_SX**

On entry, at least one element of **sx** is negative.

**NE\_POS\_SX**

On entry, no element of **sx** is positive.

**NE\_SW\_LT\_ONE**

On entry, the sum of weights is less than 1.0.

**NE\_VAR\_EQ\_ZERO**

A variable has zero variance.

At least one variable has zero variance. In this case **v** and **std** are as calculated, but **r** will contain zero for any correlation involving a variable with zero variance.

**NE\_ALLOC\_FAIL**

Memory allocation failed.

**6. Further Comments**

Correlation coefficients based on ranks can be computed using `nag_ken_spe_corr_coeff` (g02brc).

**6.1. Accuracy**

For a discussion of the accuracy of the one pass algorithm see Chan *et al* (1982) and West (1979).

**6.2. References**

Chan T F, Golub G H and Leveque R J (1982) Updating Formulae and a Pairwise Algorithm for Computing Sample Variances. *Computat. Physica-Verlag*.

West D H D (1979) Updating Mean and Variance Estimates: An Improved Method. *Comm. ACM* **22** (9) 532–535.

**7. See Also**

`nag_ken_spe_corr_coeff` (g02brc)

**8. Example**

A program to calculate the means, standard deviations, variance-covariance matrix and a matrix of Pearson product-moment correlation coefficients for a set of 3 observations of 3 variables.

**8.1. Program Text**

```

/* nag_corr_cov(g02bxc) Example Program
 *
 * Copyright 1994 Numerical Algorithms Group.
 *
 * Mark 3, 1994.
 */

#include <nag.h>
#include <stdio.h>
#include <nag_stdlib.h>
#include <nagg02.h>

```

```

#define NMAX 5
#define MMAX 5
#define TDX MMAX
#define TDV MMAX
#define TDR MMAX
main()
{
    double x[NMAX][TDX], r[MMAX][TDR], v[MMAX][TDV];
    double wt[NMAX], *wtptr;
    double sw, wmean[MMAX], std[MMAX];
    Integer i, j, n, m;
    char w;
    Integer tdx, tdr, tdv;
    Integer test;

    Vprintf("g02bxc Example Program Results\n");

    /* Skip heading in data file */
    Vscanf("%*[^\\n]");
    tdx = TDX;
    tdr = TDR;
    tdv = TDV;
    test = 0;

    while ((scanf("%ld%ld %c",&m,&n, &w) != EOF))
    {
        if (m>=1 && m<=MMAX && n>=1 && n<=NMAX)
        {
            for(i=0; i<n; i++)
                Vscanf("%lf",&wt[i]);
            for(i=0; i<n; i++)
                for(j=0; j<m; j++)
                    Vscanf("%lf",&x[i][j]);

            if (w == 'w')
                wtptr = wt;
            else
                wtptr = (double *)0;

            g02bxc(n, m, (double *)x, tdx, (Integer *)0, wtptr, &sw, wmean, std,
                (double *)r, tdr, (double *)v, tdv, NAGERR_DEFAULT);

            if (wtptr)
                Vprintf("\\nCase %ld --- Using weights\\n", ++test);
            else
                Vprintf("\\nCase %ld --- Not using weights\\n", ++test);

            Vprintf("\\nInput data\\n");
            for(i=0; i<n; i++)
                Vprintf("%6.1f%6.1f%6.1f%6.1f\\n",x[i][0],x[i][1],x[i][2],wt[i]);

            Vprintf("\\n");
            Vprintf("Sample means.\\n");
            for(i=0; i<m; i++)
                Vprintf("%6.1f\\n",wmean[i]);
            Vprintf("\\nStandard deviation.\\n");
            for(i=0; i<m; i++)
                Vprintf("%6.1f\\n",std[i]);

            Vprintf("\\nCorrelation matrix.\\n");
            for(i=0; i<m; i++)
            {
                for(j=0; j<m; j++)
                    Vprintf(" %7.4f ",r[i][j]);
                Vprintf("\\n");
            }

            Vprintf("\\nVariance matrix.\\n");
            for(i=0; i<m; i++)
            {

```

```

        for(j=0; j<m; j++)
            Vprintf(" %7.3f ",v[i][j]);
            Vprintf("\n");
        }
        Vprintf("\nSum of weights %6.1f\n", sw);
    }
    else
    {
        Vfprintf(stderr, "One or both of m and n are out of range:\n
m = %-3ld while n = %-3ld\n", m, n);
        exit(EXIT_FAILURE);
    }
}
exit(EXIT_SUCCESS);
}

```

## 8.2. Program Data

g02bxc Example Program Data

```

3 3 w
 9.1231  3.7011  4.5230
 0.9310  0.0900  0.8870
 0.0009  0.0099  0.0999
 0.1300  1.3070  0.3700

```

```

3 3 w
 0.1300  1.3070  0.3700
 9.1231  3.7011  4.5230
 0.9310  0.0900  0.8870
 0.0009  0.0099  0.0999

```

```

3 3 u
 0.717  9.370  0.013
 1.119  0.133  9.700
11.100 23.510 11.117
 0.900  9.013  8.710

```

```

3 3 w
 0.717 19.370  0.013
 1.119  0.133  9.700
11.100 23.510 11.117
 0.900  9.013 78.710

```

```

3 3 u
 0.717 19.370  0.013
 1.119  0.133  9.700
11.100  3.510 13.117
 0.900  0.013 78.710

```

```

3 3 w
 0.717 19.370  0.913
 1.119  0.133  9.700
17.100 93.510 13.117
30.900  0.013 78.710

```

## 8.3. Program Results

g02bxc Example Program Results

Case 1 --- Using weights

Input data

```

 0.9  0.1  0.9  9.1
 0.0  0.0  0.1  3.7
 0.1  1.3  0.4  4.5

```

Sample means.

```

 0.5
 0.4
 0.6

```

Standard deviation.

0.4  
0.6  
0.3

Correlation matrix.

1.0000	-0.4932	0.9839
-0.4932	1.0000	-0.3298
0.9839	-0.3298	1.0000

Variance matrix.

0.197	-0.123	0.149
-0.123	0.316	-0.063
0.149	-0.063	0.117

Sum of weights 17.3

Case 2 --- Using weights

Input data

9.1	3.7	4.5	0.1
0.9	0.1	0.9	1.3
0.0	0.0	0.1	0.4

Sample means.

1.3  
0.3  
1.0

Standard deviation.

3.3  
1.4  
1.5

Correlation matrix.

1.0000	0.9908	0.9903
0.9908	1.0000	0.9624
0.9903	0.9624	1.0000

Variance matrix.

10.851	4.582	5.044
4.582	1.971	2.089
5.044	2.089	2.391

Sum of weights 1.8

Case 3 --- Not using weights

Input data

1.1	0.1	9.7	0.7
11.1	23.5	11.1	9.4
0.9	9.0	8.7	0.0

Sample means.

4.4  
10.9  
9.8

Standard deviation.

5.8  
11.8  
1.2

Correlation matrix.

1.0000	0.9193	0.9200
0.9193	1.0000	0.6915
0.9200	0.6915	1.0000

Variance matrix.  
 33.951    63.208    6.485  
 63.208    139.250    9.871  
 6.485    9.871    1.464

Sum of weights    3.0

Case 4 --- Using weights

Input data  
 1.1   0.1   9.7   0.7  
 11.1 23.5 11.1 19.4  
 0.9   9.0 78.7   0.0

Sample means.  
 10.7  
 22.7  
 11.1

Standard deviation.  
 1.9  
 4.5  
 1.8

Correlation matrix.  
 1.0000    0.9985    0.0173  
 0.9985    1.0000    0.0716  
 0.0173    0.0716    1.0000

Variance matrix.  
 3.672    8.538    0.059  
 8.538    19.909    0.570  
 0.059    0.570    3.185

Sum of weights    20.1

Case 5 --- Not using weights

Input data  
 1.1   0.1   9.7   0.7  
 11.1   3.5 13.1 19.4  
 0.9   0.0 78.7   0.0

Sample means.  
 4.4  
 1.2  
 33.8

Standard deviation.  
 5.8  
 2.0  
 38.9

Correlation matrix.  
 1.0000    0.9999    -0.4781  
 0.9999    1.0000    -0.4881  
 -0.4781    -0.4881    1.0000

Variance matrix.  
 33.951    11.567    -108.343  
 11.567    3.941    -37.687  
 -108.343    -37.687    1512.750

Sum of weights    3.0

Case 6 --- Using weights

Input data

1.1	0.1	9.7	0.7
17.1	93.5	13.1	19.4
30.9	0.0	78.7	0.9

Sample means.

17.2
86.3
15.9

Standard deviation.

4.2
25.6
13.7

Correlation matrix.

1.0000	-0.0461	0.7426
-0.0461	1.0000	-0.7033
0.7426	-0.7033	1.0000

Variance matrix.

17.846	-4.989	43.123
-4.989	656.407	-247.692
43.123	-247.692	188.970

Sum of weights 21.0

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